

$\frac{\mathbb{Z}}{2\mathbb{Z}} \times \frac{\mathbb{Z}}{2\mathbb{Z}} = G$ مثال

$G = \{ (\bar{0}, \bar{0}), (\bar{0}, \bar{1}), (\bar{1}, \bar{0}), (\bar{1}, \bar{1}) \}$

إنه ($G, +$) زمرة تبديلية حيث
 $(\bar{a}_1, \bar{b}_1) + (\bar{a}_2, \bar{b}_2) = (\bar{a}_1 + \bar{a}_2, \bar{b}_1 + \bar{b}_2)$

الحل

$+$	$(\bar{0}, \bar{0})$	$(\bar{0}, \bar{1})$	$(\bar{1}, \bar{0})$	$(\bar{1}, \bar{1})$
$(\bar{0}, \bar{0})$	$(\bar{0}, \bar{0})$	$(\bar{0}, \bar{1})$	$(\bar{1}, \bar{0})$	$(\bar{1}, \bar{1})$
$(\bar{0}, \bar{1})$	$(\bar{0}, \bar{1})$	$(\bar{0}, \bar{0})$	$(\bar{1}, \bar{1})$	$(\bar{1}, \bar{0})$
$(\bar{1}, \bar{0})$	$(\bar{1}, \bar{0})$	$(\bar{1}, \bar{1})$	$(\bar{0}, \bar{0})$	$(\bar{0}, \bar{1})$
$(\bar{1}, \bar{1})$	$(\bar{1}, \bar{1})$	$(\bar{1}, \bar{0})$	$(\bar{0}, \bar{1})$	$(\bar{0}, \bar{0})$

• إن $+$ متوافق تشكيل داخلي (وخصوصاً من الجذور).

• $+$ تجميعي

$((\bar{a}_1, \bar{b}_1) + (\bar{a}_2, \bar{b}_2)) + (\bar{a}_3, \bar{b}_3) =$

$= (\bar{a}_1 + \bar{a}_2, \bar{b}_1 + \bar{b}_2) + (\bar{a}_3, \bar{b}_3)$

$= ((\bar{a}_1 + \bar{a}_2) + \bar{a}_3, (\bar{b}_1 + \bar{b}_2) + \bar{b}_3)$

\mathbb{Z} تبديلي $= (\bar{a}_1 + \bar{a}_2 + \bar{a}_3, \bar{b}_1 + \bar{b}_2 + \bar{b}_3)$

$= (\bar{a}_1 + (\bar{a}_2 + \bar{a}_3), \bar{b}_1 + (\bar{b}_2 + \bar{b}_3))$

$= (\bar{a}_1, \bar{b}_1) + (\bar{a}_2 + \bar{a}_3, \bar{b}_2 + \bar{b}_3)$

$= (\bar{a}_1, \bar{b}_1) + ((\bar{a}_2, \bar{b}_2) + (\bar{a}_3, \bar{b}_3))$

• صالح الجذر $(\bar{0}, \bar{0}) \in G$ هو صيدني $+$

• تبديلي $+$ حسب توافق الجذور بالنسبة للقطر (التبديلي)

• نظير $(\bar{0}, \bar{0}) \in G$ هو $(\bar{0}, \bar{0})$

• نظير $(\bar{0}, \bar{1}) \in G$ هو $(\bar{0}, \bar{1})$

• نظير $(\bar{1}, \bar{0}) \in G$ هو $(\bar{1}, \bar{0})$

• نظير $(\bar{1}, \bar{1}) \in G$ هو $(\bar{1}, \bar{1})$

ملحوظة
 زمرة تبديلية ولكنها ليست دارة

لأنه لا يوجد عنصرين G مرتبة تساوي 4

$(G = \frac{\mathbb{Z}}{2\mathbb{Z}} \times \frac{\mathbb{Z}}{3\mathbb{Z}}, +)$ مثال

$G = \{ (\bar{0}, \bar{0}), (\bar{0}, \bar{1}), (\bar{0}, \bar{2}), (\bar{1}, \bar{0}), (\bar{1}, \bar{1}), (\bar{1}, \bar{2}) \}$

$(\bar{a}_1, \bar{b}_1) + (\bar{a}_2, \bar{b}_2) = (\bar{a}_1 + \bar{a}_2, \bar{b}_1 + \bar{b}_2)$

والحلولة : إن G تشكل زمرة تبديلية

	$(\bar{0}, \bar{0})$	$(\bar{0}, \bar{1})$	$(\bar{0}, \bar{2})$	$(\bar{1}, \bar{0})$	$(\bar{1}, \bar{1})$	$(\bar{1}, \bar{2})$
$(\bar{0}, \bar{0})$	$(\bar{0}, \bar{0})$	$(\bar{0}, \bar{1})$	$(\bar{0}, \bar{2})$	$(\bar{1}, \bar{0})$	$(\bar{1}, \bar{1})$	$(\bar{1}, \bar{2})$
$(\bar{0}, \bar{1})$	$(\bar{0}, \bar{1})$	$(\bar{0}, \bar{2})$	$(\bar{0}, \bar{0})$	$(\bar{1}, \bar{1})$	$(\bar{1}, \bar{2})$	$(\bar{1}, \bar{0})$
$(\bar{0}, \bar{2})$	$(\bar{0}, \bar{2})$	$(\bar{0}, \bar{0})$	$(\bar{0}, \bar{1})$	$(\bar{1}, \bar{2})$	$(\bar{1}, \bar{0})$	$(\bar{1}, \bar{1})$
$(\bar{1}, \bar{0})$	$(\bar{1}, \bar{0})$	$(\bar{1}, \bar{1})$	$(\bar{1}, \bar{2})$	$(\bar{0}, \bar{0})$	$(\bar{0}, \bar{1})$	$(\bar{0}, \bar{2})$
$(\bar{1}, \bar{1})$	$(\bar{1}, \bar{1})$	$(\bar{1}, \bar{2})$	$(\bar{1}, \bar{0})$	$(\bar{0}, \bar{1})$	$(\bar{0}, \bar{2})$	$(\bar{0}, \bar{0})$
$(\bar{1}, \bar{2})$	$(\bar{1}, \bar{2})$	$(\bar{1}, \bar{0})$	$(\bar{1}, \bar{1})$	$(\bar{0}, \bar{2})$	$(\bar{0}, \bar{0})$	$(\bar{0}, \bar{1})$

إنه $+$ متوافق تشكيل داخلي
 كذلك $+$ تجميعي (إنه $+$ متوافق تشكيل داخلي)

$(\bar{0}, \bar{0})$ صيدني $+$

تبديلي $+$ حسب توافق الجذور بالنسبة للقطر الرئيسي

• نظير $(\bar{0}, \bar{0}) \in G$ هو $(\bar{0}, \bar{0})$

• نظير $(\bar{0}, \bar{1}) \in G$ هو $(\bar{0}, \bar{1})$

• نظير $(\bar{0}, \bar{2}) \in G$ هو $(\bar{0}, \bar{2})$

• نظير $(\bar{1}, \bar{0}) \in G$ هو $(\bar{1}, \bar{0})$

• نظير $(\bar{1}, \bar{1}) \in G$ هو $(\bar{1}, \bar{1})$

• نظير $(\bar{1}, \bar{2}) \in G$ هو $(\bar{1}, \bar{2})$

السبب مراتب العناصر الأولية

$a = (\bar{0}, \bar{1})$

$a + (\bar{0}, \bar{0})$

$a^2 = (\bar{0}, \bar{1}) + (\bar{0}, \bar{1}) = (\bar{0}, \bar{2}) \neq (\bar{0}, \bar{0})$

$a^3 = (\bar{0}, \bar{2}) + (\bar{0}, \bar{1}) = (\bar{0}, \bar{0})$

مرتبة a تساوي 3

1 1

64

$$a = (\bar{0}, \bar{2})$$

$$a \neq (\bar{0}, \bar{0})$$

$$a^2 = (\bar{0}, \bar{2}) + (\bar{0}, \bar{2}) = (\bar{0}, \bar{4}) \neq (\bar{0}, \bar{0})$$

$$a^3 = (\bar{0}, \bar{4}) + (\bar{0}, \bar{2}) = (\bar{0}, \bar{6})$$

رتبة هذا العنصر تساوي 3

$$a = (\bar{1}, \bar{0})$$

$$a \neq (\bar{0}, \bar{0})$$

$$a^2 = (\bar{1}, \bar{0}) + (\bar{1}, \bar{0}) = (\bar{0}, \bar{0})$$

رتبة (a) هي 2

$$a = (\bar{1}, \bar{1})$$

$$a \neq (\bar{0}, \bar{0})$$

$$a^2 = (\bar{1}, \bar{1}) + (\bar{1}, \bar{1}) = (\bar{0}, \bar{2}) \neq (\bar{0}, \bar{0})$$

$$a^3 = (\bar{0}, \bar{2}) + (\bar{1}, \bar{1}) = (\bar{1}, \bar{0}) \neq (\bar{0}, \bar{0})$$

$$a^4 = (\bar{1}, \bar{0}) + (\bar{1}, \bar{1}) = (\bar{0}, \bar{1}) \neq (\bar{0}, \bar{0})$$

$$a^5 = (\bar{0}, \bar{1}) + (\bar{1}, \bar{1}) = (\bar{1}, \bar{2}) \neq (\bar{0}, \bar{0})$$

$$a^6 = (\bar{1}, \bar{2}) + (\bar{1}, \bar{1}) = (\bar{0}, \bar{0})$$

1 1

65

إذا (A, A) من المرتبة 6

إذا G مرتبة دورية

$$a = (\bar{1}, \bar{2})$$

$$a \neq (\bar{0}, \bar{0})$$

$$a^2 = (\bar{0}, \bar{4}) \neq e$$

$$a^3 = (\bar{0}, \bar{6}) \neq e$$

$$a^4 = (\bar{1}, \bar{0}) \neq e + (\bar{1}, \bar{2}) = (\bar{0}, \bar{2}) \neq e$$

$$a^5 = (\bar{0}, \bar{2}) + (\bar{1}, \bar{2}) = (\bar{1}, \bar{4}) \neq e$$

$$a^6 = (\bar{1}, \bar{4}) + (\bar{1}, \bar{2}) = (\bar{0}, \bar{0})$$

إذا رتبة a تساوي 6

التثبيت المطلوب